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Reasons for Rapid Growth in Addition Strategy Use in 1st Grade Students

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ABSTRACT

Today's quickly evolving world requires citizens to use the tools of mathematics to make informed decisions. This involves understanding the meaning and properties of mathematical operations and the ability to use them in an intuitive and flexible manner to solve problems. Working with mathematical operations begins in early primary school and establishes a basis for future work in mathematics.

Choosing the best addition strategy to compute effectively and accurately and the ability to explain their work is often challenging for young children. They often rely on familiar and safe solutions without thinking about why they chose a certain method. The purpose of this research was to gain understanding of the factors that influence students' fluency with addition, their ability to explain their work and along with that gain flexibility and confidence in mathematics.

This is action research, that involves 16 first grade students, all of whom were struggling with math at the beginning of the school year, but some of them made great gains in their ability to solve mathematics problems involving addition. The students with high performance improved in their ability to explain their actions verbally and use multiple strategies and choose appropriate and effective strategies for each task.

The aim of this research is to understand the reasons for the rapid improvement of the high performance subjects and compare it to the lower performing groups in order to help improve understanding of number operations for all students. The research involves observations and written work over a school year, and individual interviews with the students. Data about the home environment was collected from parents. The research shows that a tendency for rapid progress is due to active participation in guided class discussions and use of advanced strategies for routine tasks.

Keywords: addition strategies, calculation speed, mathematical reasoning, primary school mathematics

Introduction

In the previous year's national 3rd grade math diagnostics 25% out of all students in Latvia could not correctly add two two-digit numbers (National Centre for Education of the Republic of Latvia, 2022). What will happen with the mathematical competence of these students we can only presume, but without effective intervention it will not be satisfactory. The reason for this alarming lack of basic math operation skills undoubtedly takes root in early math, and we can assume that it is a product of teaching math through instrumental understanding rather than relational understanding (Skemp, 1987; Skemp, 2006; Boaler, 2015). What happens in the 1st grade with basic skills is fundamental and has to be looked at seriously. Since research has shown that the number of different strategies children understand and use predicts their later learning (Clements & Sarama, 2014), a closer look at the situation was needed to improve these skills.

The National Council of Teachers of Mathematics [NCTM] and other evidence-based essential math teaching practices are described below.

Teaching emphases

1. During lessons students model each new concept (or whenever it is necessary for them) with manipulatives, visuals and descriptive representations and make connections between them (NCTM, 2014; Smith et al., 2017). Five types of representations are used to gain understanding of a concept: physical, visual, contextual, verbal and symbolic (Cramer, 2003; Huinker, 2015; Huinker & Bill, 2017; Lesh et al., 1987). The trajectory of representations on how to build understanding of each concept is from concrete models, through pictorial representations and finally introducing abstract form (Bruner & Bruner, 1966; Lesh & Doerr, 2003; Hurrel, 2021).
2. To develop number and operational sense:
   - Deliberate practice to conceptually subitize (Clements & Sarama, 2014; Walle et al., 2018) using subgroups of dots to learn how to reason about quantities in flexible ways since "subitizing supports children's development of number sense and operation sense; it engages children in decomposing and composing numbers as a basis for reasoning strategies with basic facts for addition" (NCTM, 2020, 84).
   - Learn basic number combinations through understanding not memorization (Clements & Sarama, 2014; NCTM, 2020).
3. Explain and justify: the teacher focuses on letting students explain their thinking and compare it to the way their peers are thinking. Students know that in this classroom different approaches are valued, so they try to think of alternative (maybe not yet the most effective) ways of reasoning (NCTM, 2014; Smith et al., 2017). Students learn to judge which ways are the most

effective or most comfortable for them. "Classroom discussion must move beyond children providing short answers to direct questions toward explaining and justifying their reasoning and answers, which not only benefits individual children's learning but also contributes to the shared learning of the classroom community" (NCTM, 2020, 74).

4. Help students notice and use mathematical structures (NCTM, 2020) by organizing the tasks so there are visible patterns (Fan et al., 2015). Noticing how a previous problem is similar to another one can help to figure out the new one. Practice by looking for patterns in 100's chart and other number and figure arrangements. By noticing structures children see that mathematics make sense and that their ideas are valued and worth exploring and discussing. Students learn to observe, make educated guesses and generalizations.
5. Teach that there are specific different addition strategies (Clements & Sarama, 2014; Walle et al., 2018), see the possible strategies with clarifications are provided by the authors in Table 1.

Table 1. Addition strategies

Represents addends with manipulatives (fingers, base ten blocks, two colored counters and others) and counts	▸ from 1 ▸ count on from one addend
Counting on	▸ counting on from the first addend ▸ counting on from the largest addend
Making a 10	▸ e.g. 8 + 7 = 8 + 2 + 5
Memorized addition facts	
Reasoning from known addition facts	
Adding tens and adding ones	▸ with manipulatives ▸ without manipulatives
Mix of strategies	▸ when adding two-digit numbers, knowing the ten's sum, adding the ones by counting on ▸ adding tens by counting on, and then adding the ones by making a ten ▸ etc.
Personal strategy	▸ counting by twos ▸ 5 as a benchmark, e.g. 8+7 = 5 + 5 + 3 + 2 ▸ etc.

6. Build procedural fluency from conceptual understanding (NCTM, 2014; Walle et al., 2018, Burns et al., 2015; Smith et al., 2017).

All of these are evidence-based effective practices and many students benefit from them greatly, but still others continue to struggle with basic operations. The authors searched for an answer as to why this is so by posing the following questions:

1. Does multiple strategy use indicate an overall better performance in addition?
2. What might be the reasons for rapid growth in addition strategy use?
3. What does a student's speed at solving addition problems tell us about their addition skills?

Methodology

This is an action research that was conducted by an educator with the guidance of two researchers. This form of research was chosen because the impact for the teacher and students is immediate and corresponds to the specific setting and problem (Efron & Ravid, 2019). The research originated with a math teacher who implemented into her teaching all of the mathematics teaching practices described in the subsection "teaching emphasis". Still, some of her 1st grade students did not show satisfactory addition skills.

In the beginning of the 2021./2022. school year the first grade math teacher made an observation that 16 out of 48 of her new students might have difficulties in math in the coming year based on their math skills and knowledge in September. At the end of the school year from May 24th to 27th a rigorous diagnostic of their addition skills was conducted. During the assessment the teacher offered 11 addition problems one at a time for the pupil to solve, using manipulatives or paper and pen as needed. The process was videotaped and analyzed (see Figures 1 and 2).

Figure 1. A screenshot from the diagnostic process video

Figure 2. The setting

Permissions to videotape, publish pictures and participate in the research were obtained from the parents of the students. The problems were later divided in two parts: 6 of the addition problems had been covered in the 1st year curriculum (consisting mostly of one digit addends). These were **routine tasks**. Five of the problems dealt with two or three digit numbers or problems that were not taught yet. These were **challenging tasks**, to test the childs' ability to make connections and transfer his skill to a new settings. The tasks were designed so the pupils could demonstrate different strategies (see Table 2).

Table 2. Addition problem categories

routine tasks						challanging tasks				
7 + 2	3 + 8	8 + 5	8 + 8	8 + 9	24 + 6	17 + 6	33 + 46	15 + 17 + 15	12 + 28	57 + 36

Note. Highlighted are problems not covered in the 1st grade program.

Two indicators were identified; whether a child was able to make connections and whether they chose an effective strategy. Based on their performance on the diagnostic task (see Figure 3), the children were divided in three groups: children with low performance (if both indicators were below 50%) in their math skills (n = 7), average performance (n = 5) (if at least one indicator was 50%) and high performance (n = 4) (if both indicators were above 50%). The first indicator showed students' ability to solve addition problems that were not taught in class. This indicated the ability to make connections and transfer knowledge. If a child was able to solve 28 + 12, based on the skill to add one digit addends so a new ten is formed, it was decided that he can make some connections; if a child could solve 57 + 36, a more complicated situation with regrouping, then it was categorized that the child can make connections. The second indicator was the students' effective use of strategies. A strategy was deemed effective if 3 requirements were met: first, the strategy is objectively effective; second, the child was fluent in using the strategy; and third, the answer was correct.

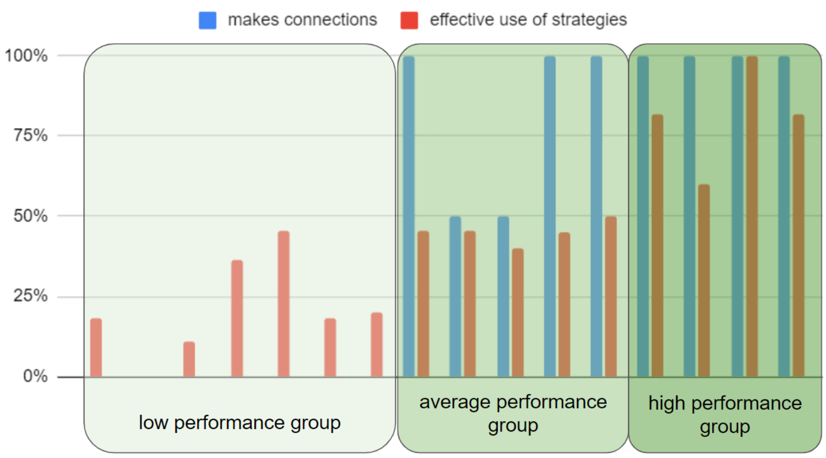

Figure 3. Grouping principle

Results

1. Does multiple strategy use indicate an overall better performance in addition?

In the Table 3 we can see a tendency: the higher the performance group, the higher the percentage of correct answers and better the ability to explain one's reasoning. It is not clear if because of the ability to explain one's reasoning the child can effectively use strategies or the other way around. Regardless, we can reason that it is more likely that if a pupil can explain his reasoning, he will be able to make connections, be more precise in his calculations and use strategies effectively.

Table 3. Average student demonstrated skills in the research setting

Group	Low performance	Middle performance	High performance
precision	72%	77%	98%
ability to explain one's reasoning	36%	40%	88%

Note. Precision refers to correctly solved addition tasks. The ability to explain one's reasoning indicates whether the student could recall and understandably explain or show his addition strategy.

Figure 4 shows how many different strategies children used to solve the 11 given addition sentences. In this research, the high performance group used more strategies than the low. The reasons for low strategy count might differ.

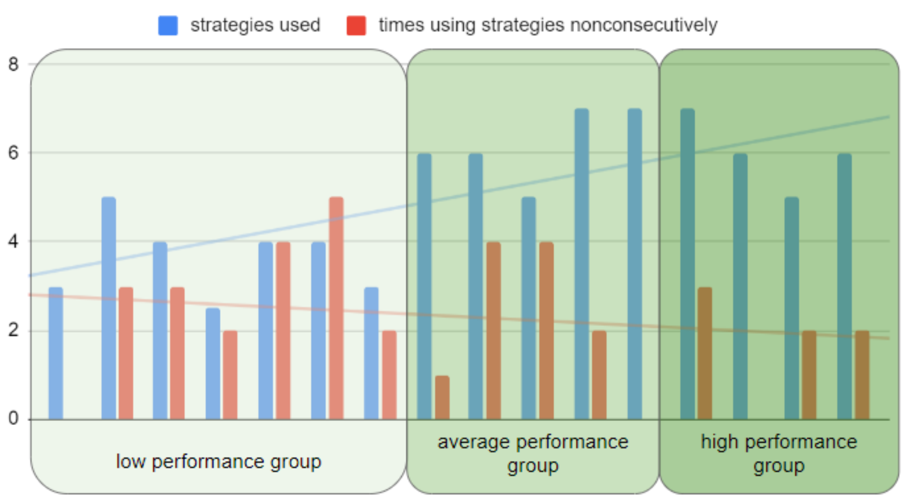

Figure 4. Strategy use

For example, the 1st student felt safe with using the counting-on strategy and used it with various success for almost every problem. But, if it is because she felt safe with it and for that reason did not learn new strategies provided in lessons, we can only speculate. The 2nd student had a poor understanding of place value, that did not allow him to use strategies that relied on this knowledge. The 3th student tried to use strategies that she did not really understand why they worked, so fell back to using the simplier strategy of modeling: make the numbers out of manipulatives and count.

The other data visible in the graph shows how many times the students used a nonconsecutive strategy. This in the design phase of the research was thought to show the flexibility of strategy use since the child did not use the previous strategy, but chose another from his strategies. We can see that more "jumping" in between strategies happened in the low and average group. We can reason that since the higher group uses more strategies it is less likely they will use any previously used strategy. There is another possibility that occurred looking at the data. There are children who, when facing a math problem, do not mentally look through their arsenal of tools (strategies) and then decide for the best one in the situation. Instead they start doing something without giving it a thought if it is effective or the best strategy. They are just keen to get a result, that led to chaotic "jumping" through their strategies without deliberation. This might be the story of the 5th and 6th student.

2. What might be the reasons for rapid growth in addition strategy use?

Children in the higher performance group seemed to have more confidence in math (see Table 4). The confidence was measured from the parent questionnaire and from student comments. This evaluation is by nature subjective. If a student has low confidence he was scored 0, if average confidence: 0.5, if great confidence: 1.

In the second row of Table 4 we can see that it is more likely that a child will have higher performance at the end of a school year if he participates in class discussion. This means he uses math language, explains his thinking to others, listens to other solutions and all in all is an active participant in learning. If a student participated in classroom discussion only if asked directly, he was scored 0, if participate rarely: 0.5, if participated often: 1.

Table 4. Teacher observations in classroom settings. The average scores of each group.

Group	Low performance	Average performance	High performance
participation in class discussion	0.36	0.6	0.88
confidence in math	0.5	0.6	0.88

Note. Both indicators were measured in a scale 0 to 1.

Gathered data from the home environment showed a childs' additional math learning at home. The use of applications and games related to mathematics did not show any correlation to skills. Some children from the low, average and high performance group did extra learning at home and some did not. Contrary to what was predicted in this sample, school attendance data showed no relationship to performance level (see Table 5), although in individual cases both extra learning (10th &14th student) or high attendance might be one of the reasons for higher performance (or low attendance for low performance: 2nd, 3d & 6th student).

Table 5. School atendance

Group	Low performance							Average performance					High performance			
student number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
missed lessons	101	134	160	47	63	142	36	129	83	158	77	62	61	189	117	108
average	97.57							101.80					118.75			

Note. The number of overall missed lessons including math lessons.

3. What does addition sentence solving speed tell about addition skills of a pupil?

When analyzing the videos, the time for completing each task was measured, from the moment the task was given until the student gave the answer. The overall average speed was higher for the high performance group (see Table 6). The authors would like to point out that the difference between time needed to solve routine and challenging problems is significantly higher for the low group. One of the reasons we observed was that the routine tasks with smaller numbers allowed more basic strategies (representing addends with manipulatives and counting them or counting on), that were either unsuccessful or very time consuming for the challenging tasks.

Table 6. Average addition problem solving speed in seconds

Group	Low performance	Average performance	High performance
routine tasks	21.13	21.74	12.22
challenging tasks	83. 42	52.02	36.52
difference	62.29	30.28	24.29

When looking at separate situations (see Figure 5), there were children from the low performance group who solved the easy tasks even faster (up to 12 sec.) on average than children from the high group. These same children solved the hard tasks significantly slower. The reason why the high performance group children in this research solved easy problems slower could be because they used more advanced strategies (e.g. making ten, reasoning from a known sum) in situations where the basic strategies may give faster answers, but in the long run the children using the more advanced strategies in easy tasks and taking more time, had better understanding in using them in general and were comparatively faster in challenging tasks.

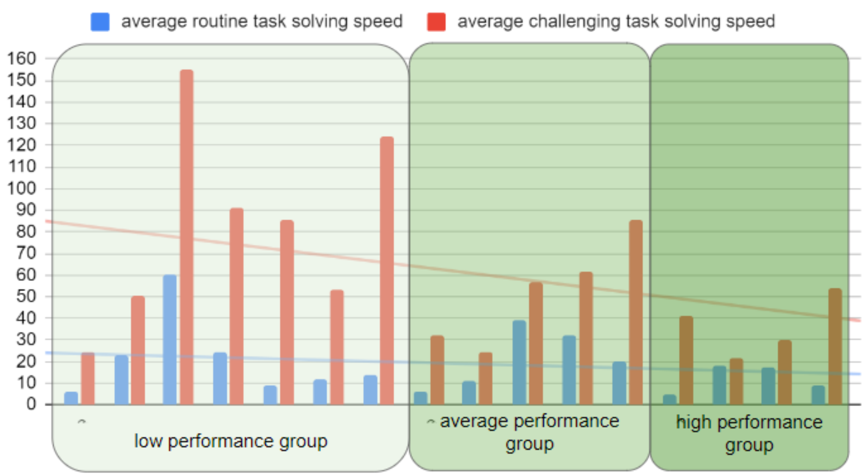

Figure 5. Average routine and challenging task solving speed in seconds. Individual students

Discussion

Our research confirmed that multiple strategy knowledge and use is advantageous in early math (Clements & Sarama, 2014), because the higher performance group used more strategies. Nonconsecutive use of strategies is not an unequivocal indicator, and is dependent on the order of the tasks, not just selection of most effective strategy and is not a certain indicator of fluency in this research. Understanding and using multiple strategies is important not just in the early years of learning math, but predicts their later learning (Clements & Sarama, 2014).

Not surprisingly this research confirmed that it is more likely that a child will have confidence in math if he is in the higher performance group, since we tend to like the things we are good at (Kohn, 2018). It is more likely a child will have better performance in mathematics if he actively participates in classroom discussions and peer discussions, since we understand better the concepts we talk about (Harlen & Qualter, 2018). School attendance and home environment were significant only in individual cases, but in this research sample no general tendencies were illuminated.

The most exciting observations or conclusion from this research was when looking at speed. As Jo Boaler (2015) pointed out math is not about speed, which is a widespread assumption, and if speed is an incessant part of math teaching and learning we can overlook slower and deeper thinkers. The data in this research shows that overlooking slow thinkers is not the only problem with speed in math, but can give teachers false security in children's math competence. This illuminates the necessity to know how a student gets to an answer, not just if it is correct.

For the classroom teacher this indicated:

1. A teacher observing that a child produces fast and precise answers might conclude that they do not need any intervention and are overlooked in the sense that they do not develop advanced strategy use and reasoning skills, so causing problems later on. Easy addition problems do not demand reasoning, only applying known algorithms. A child might lose the opportunity to master more advanced strategies if they are allowed to use only basic strategies.
2. Speed should not be the main component in mathematics, but rather the "how" and "why" and the efforts to find the most effective strategies and explaining one's reasoning.

Further research would be beneficial in discovering why some students do not use advanced strategies, and how to lead them to fluency in different strategies, because research shows that the use of more strategies is beneficial.

Additional data is needed to specify the reasons why some students become high and some remain low performers.

Conclusions

Most of the students who were in the low performance group had different gaps in their skills or knowledge that did not allow them to do the challenging addition tasks. For example, one of the students had not developed understanding of place value. Another child tried to use strategies he did not understand and another could not break the habit of adding by counting by ones and other made mistakes in rote counting. Each of these problems or gaps require different intervention strategies.

The authors see a possible two step solution:

1. Diagnostic can not just rely on correct answers, as the teacher can not see the thinking process or strategy. For this kind of diagnostic preliminary

work in the classroom is necessary: children must have been exposed to a teacher modeling their thinking and experience and practice explaining one's reasoning and learning to understand their classmate's strategies and thinking.

2. Early and continuous individualized intervention is necessary that is focused on the specific knowledge or skill gaps of each student. The emphasis is on *continuous*. It should be performed until the gap is fully closed, because some interventions and small after class individualized work did occur for the students in this research, but not systematically. The problem in the early math classroom is that the lack of fluency can be due to a lack of different skills for various students. The basic skills do not develop fast, time and regularity is necessary but not for all students and not for the same skill.

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