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Changes in the Mathematics Curriculum for Grades 10–12 in Latvia

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ABSTRACT

In the school year 2021/2022 in all schools and for all grades in Latvia ends the gradual transition from the previous mathematics standard (that was used since year 2008) to the new mathematics standard. This standard is developed within the ESF project "Competence Approach to Curriculum (School 2030)". The mathematics subject in the secondary school is divided into two parts: Mathematics I (optimal level for every secondary school student) and Mathematics II (advanced level for secondary school students who plan to study exact sciences at the university). The final exam also is different and will focus not only on solving problems but also on the correct use of mathematical language and justification. These changes also have effect on the learning process, now emphasis is on the competency-based learning that will provide pupils with the knowledge, skills and attitudes needed for modern life.
The aim of our study is to compare standards, curricula, and final exams to analyse the differences in mathematics content for grades 10–12. The method used in this article is document analysis as documentary research as well as analysis of pupils' results in final mathematics exams.
As the education system in Latvia is in the process of transition, it is important to understand how the changes might affect pupils' knowledge and skills in mathematics.

Keywords: competency-based education, education system in Latvia, mathematics curriculum, mathematics for Grade 10–12, mathematics final exam, project School2030, secondary education standard

Introduction

Mathematics education has been changing and developing in Latvia as everything else over the last twenty years of independence. The changes affected educational programmes, teaching methods etc. Basic educational attitudes in

Latvia changed from traditional (heritage of soviet education) to progressive education.

Currently, education systems in every country are different, but there are international studies that evaluate the academic performance of pupils, so that each country can compare the performance of its pupils with others and decide on the necessary changes.

For example, PISA is an international survey which is aimed at the evaluation of education systems worldwide by testing skills and knowledge of 15-year-old pupils. It is conducted every three years. One of areas in which students are assessed in this study, is mathematics. The mean score in mathematics from PISA 2018 for Latvia in mathematics was 496, i.e., pupils in Latvia scored higher result than the OECD average (489) in mathematics (PISA 2018 Results. Combined Executive Summaries. Volume I, II & III, 2019). For comparison, the two other Baltic countries with a similar history have the following mean scores in mathematics: Lithuania 481; Estonia 523. Mean mathematics performance was significantly higher in PISA 2018 than in PISA 2015, but when considering the entire 2003-2018 period, mathematics performance appeared to oscillate around a stable mean, with no clear direction of change; a more consistently positive trend is observed amongst the lowest-achieving students in mathematics, narrowing the gap between those and higher-achieving students to some extent (Country Note. PISA 2018 Results. Latvia, 2019).

TIMSS is an international assessment of mathematics and science, it is conducted every four years for Grade 4 and Grade 8. Average mathematics scale score for Latvia (Grade 4) in 2019 was 546, TIMSS scale centerpoint was 500, for comparison, Lithuania 542, Estonia did not take part in this study (Mullis et al., 2020). There are no results for Grade 8 pupils.

Unfortunately, both studies concern pupils in primary education. There are no international studies for pupils in secondary education. We can internationally compare the performance of secondary school pupils in mathematics through mathematical olympiads. For example, relative ranking for Latvia in last three years in the International Mathematical Olympiad is respectively 36.54%, 55.66%, 39.81%; for Lithuania 49.04%, 29.25%, 54.37%; for Estonia 46.15%, 53.77%, 52.43% (International Mathematical Olympiad. Latvia. Team Results, 2022). Unfortunately, these results do not reflect knowledge and skills that can be acquired in mathematics lessons.

The other way how to compare different education systems in countries to gain knowledge about possibilities for improvement is to compare curricula. Note that this is not the only way, for example, different levels of comparison in educational studies are described in (Bray & Thomas, 1995).

There are several research studies describing or comparing education systems in different countries. For example, Moravcova et al. (2019), Kácovský et al. (2022),

Prendergast & Treacy (2017), Demirtaş et al. (2015), Norvaiša (2019). Curricula from other countries are often not available in English, making it difficult to compare them. However, we do not know any publication that compares a mathematics curriculum of Latvia with other countries or describe changes in mathematics curriculum in Latvia.

The aim of our study is to describe and compare standards, curricula, and final exams to analyse the differences in mathematics content for grades 10–12, as well as to investigate the immediate effect (if any) that this change has on pupils' final exam results. This will be done through analysing the results of two types (the old one and the new one) of final exam in 2022.

In Latvia the following levels of education are specified:

1) pre-school education,
2) basic education (Grades 1–9, age from 7 to 16),
3) secondary education (Grades 10–12, age from 16 to 19),
4) higher education (Education Law, 1998).

More about system of education in Latvia see (System of education, 2022).

Curriculum is a broad term. In our research, we analyse only official national documents of intended curriculum for secondary education in which we can find learning content, time allotment, educational goals, and objectives.

In Latvia, the official document that sets out requirements for secondary education is Republic of Latvia Cabinet of Ministers Regulation No. 416 (Regulations Regarding the State General Secondary Education Standard and Model General Secondary Education Programmes, 2019). Since 2020, the new standard gradually is implemented in different grade groups. This standard is developed as part of the National Centre of Education ESF project Nr.8.3.1.1./16/I/002 "Competence Approach to Curriculum (School 2030)" (About Project, 2022). These changes affect educational institutions from pre-school to secondary school, most significant changes in mathematics are in Grades 10–12, that will be discussed in this article.

In order to make it easier for teachers to fulfil the requirements of the law, in Latvia a sample of curriculum for mathematics is centrally developed and it is recommended to use it when organizing the mathematics learning process. The sample curriculum is only a recommendation, so teachers can develop their own curriculum accordingly to the subject standard. In this article we analyse a curriculum for the optimal level and a curriculum for the highest level: (Mathematics I. Sample curriculum of the optimal course for general secondary education, 2020) and (Mathematics II. Sample advanced course curriculum for general secondary education, 2020). Curriculum for the general level course Mathematics will not be analysed in the article.

Methodology

The research subject of this work is the changes in the mathematics curriculum for Grades 10-12 in Latvia. The study aims to overview the current situation of education system and teaching mathematics. The method used in this article is document analysis as documentary research where we analyse general and specific changes in mathematics curricula compared to the previous mathematics standard and curricula that were introduced in 2008. The focus was on the optimal and higher level courses. Also, we analyse pupils' results in the mathematics final exam.

Results

This section is divided into four parts. We start with a brief history of education system in Latvia. In the second and third part we describe the general and specific changes of the subject Mathematics in the secondary education system in Latvia. As final part we analyse pupils' results in mathematics final exam.

A Brief History of the Mathematics Education System in Latvia

In 1991, Latvia regained its independence, therefore there was a need for new educational system. As a result, in 1991, the Law of Education was adopted.

In 1995, with the conformation of the concept of Latvian Education the reform of the educational content was started. In 1998, the first national basic education standard was approved, as well as a new Law on Education was adopted. A year later, in 1999, the law on general education and the law on vocational education were introduced. The reform lasted until 2002 when all grades were taught by the new standard. During this period a national examination system was established, which provided national examinations in the grade 3, 6, 9 and 12 and final centralized examinations at the end of general secondary education programs (Andersone, 2022).

In 2008 a new curriculum with reduced content of mathematics for secondary school (grade 10–12) was developed. Based on this curriculum new national education standards were approved for primary education (2014) and for general secondary education (2013). Subject curricula and educational standards based on the competency approach are being developed and improved from 2012 to 2017. New educational standards and curricula are immediately introduced a system of national tests and centralized examinations that encourage to faster introduce the innovations (Andersone, 2022).

During this period the main changes in the content of mathematics were the removal of complex numbers, polynomial division, elements of mathematical analysis and elements of analytic geometry from the curriculum. Until 2002 these topics were included in advanced mathematics courses.

In 2020 significant changes are again introduced in mathematics content. As a result, a pupil can choose to obtain mathematics in advanced level that includes for example elements of mathematical analyses and elements of analytic geometry.

Secondary school education now is divided in three levels – general, optimal and highest. (See Figure 1 as example for mathematics). For each level the intended results to be achieved by a pupil have been formulated, see (Regulations Regarding the State General Secondary Education Standard and Model General Secondary Education Programmes, 2019).

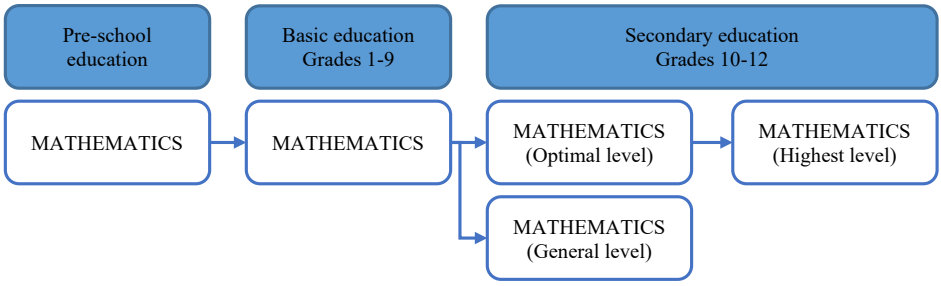

Figure 1. Three levels of secondary school education

On the general level a pupil must solve problems in familiar situations, supplement, generalise, and systematise the knowledge, understanding, and skills acquired in the basic education. On the general level of acquisition, it is mandatory to acquire a part of the content of each field of study according to the standard of general secondary education. This level is more intended for pupil who have chosen secondary education with professional orientation in some direction (for example, arts, music, cook, tailor).

On the optimal level a pupil strengthens skills to plan and implement independent action of cognition and problem-solving, identifies and solves problems in simple, unfamiliar situations, establishes deepened conceptual understanding in the field of study with interdisciplinary elements, demonstrates complex skills, acquires product creation experience. Acquisition of the study content on the optimal level is a pre-condition to study on the highest level and is important for comprehensive general secondary education.

On the highest level the pupil intentionally, responsibly, creatively, and independently plans and supervises his cognitive activity, independently solves problems in unfamiliar, complex situations, establishes a deep conceptual understanding in the field of study, and recognises interdisciplinary regularities. On

this level the study content is advanced, expanded, and significant for the further studies at the universities.

All study subjects in the secondary school are divided into courses. There are two or three courses for each subject, these courses are named Subject, Subject I and Subject II (Sample curricula in secondary education, 2020). For the general level are courses Subject and some Subject I courses (for example, pupil must learn course Mathematics and course Latvian I, as well as pass the appropriate exams). The general level is separated from secondary education and will not be analysed in this article.

For the optimal level are intended courses Subject I, but for the highest level courses Subject II (for example, pupil learns Mathematics I on the optimal level and Mathematics II on the highest level). A pupil must choose to acquire at least three courses in the highest level and pass at least two appropriate highest level exams. To graduate a secondary school pupil must pass at least four exams, including exam in Latvian (native language), foreign language and mathematics. All exams must be at least in the optimal level and at least two exams must be in the highest level.

General Changes in Mathematics for Grades 10–12 in Latvia

Since there are crucial general changes in the secondary education system, it also influences the subject curricula and teaching process organization. Previously in secondary school pupils learned mathematics all three years (grade 10–12) and there was the same final exam, regardless of the number of math classes that could have differed from 4 to 8 hours per week. Accordingly, to the new standard a school can choose what level courses it will provide to pupils and how the learning process will be organized. There are four main options how mathematics can be taught (see Figure 2). In the most cases schools choose to teach course Mathematics I in Grade 10 and Grade 11 (each year 6 hours per week), after completing this course, pupils take an optimal level exam.

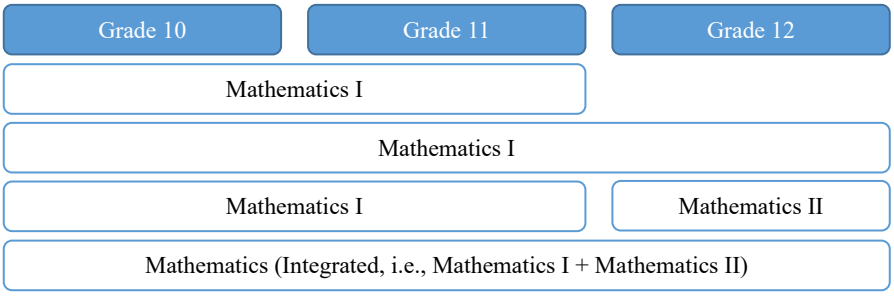

Figure 2. Course Mathematics I and Mathematics II learning organization in schools

The knowledge acquired in the course Mathematics I (the optimal level) is sufficient for further studies in areas where mathematics is not a profiling course. Thus, if school provides and pupil chooses the course Mathematics II (the highest level in which a pupil acquires knowledge that is essential for further studies where mathematics is necessary, for example, physics, computer science) is taught in Grade 12 (8 hours per week). At the end of grade 12 a pupil can take a highest-level exam if he needs it. Other option is to teach course Mathematics I in grades 10–12 choosing an appropriate number of lessons per week and taking an optimal level exam at the end of Grade 12. There are some schools (mostly gymnasiums) that teach both mathematics courses in an integrated manner without diving the content of mathematics into two parts.

Since the project School2030 developed the standard from pre-school till secondary school education then the standard of mathematics for secondary education continues the structure of basic education and also has six main parts (Regulations Regarding the State General Secondary Education Standard and Model General Secondary Education Programmes, 2019):

- the language of mathematics,
- strategies and reasoning characteristic to mathematics,
- numbers, operations on numbers,
- elements of algebra and functions,
- data and elements of statistics,
- shapes.

The approach to learning process itself also has changed (see Figure 3). If before the 2000s a teacher was more like a lecturer who gave ready knowledge and facts to pupils and pupils mostly solved tasks by the given algorithm, then now a teacher must become a consultant that helps pupils to construct new knowledge themselves and apply it in new situations.

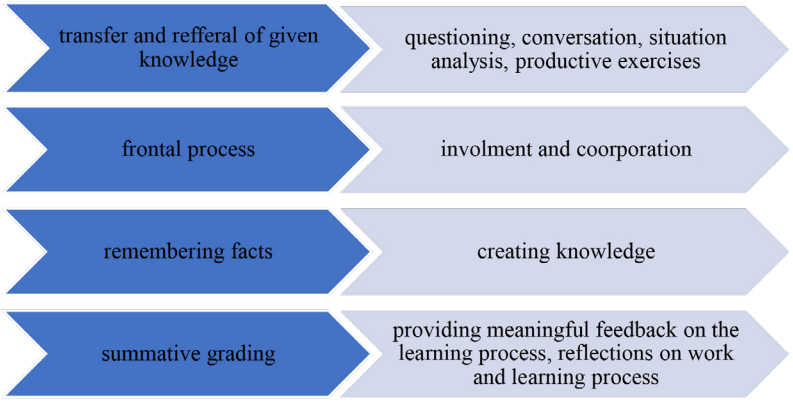

Figure 3. Main Changes in the Teaching and Learning Process

Thus, the learning process has changed from transfer and referral of knowledge to creating new knowledge by questioning and exploring. The frontal teaching process changes to process where pupils take active involvement and corporation. The accents are on using and creating knowledge as opposed the previous situation where emphasis was more on remembering specific facts as a goal of a study subject. Understanding a mathematical concept is put primary to practising, which is necessary but secondary. The exam results in next few years will show how these changes have affected students' mathematical knowledge since remembering mathematical facts and formulas are necessary to be able to solve complex problems and create new knowledge.

Evaluation system also has changed, now a teacher must set learning outcomes and give meaningful feedback on the process of learning and further development, as well as pupils should be encouraged to reflect on their learning and thinking.

Continuity, systematicity and integrity are the guiding principles behind the new standard and curriculum. Consequently, the course Mathematics II is built as a logical continuation for the course Mathematics I (see Table 1). In order to create a transparent and content-successive relationship between these courses, their content is organized in modules, for example, the module "Analytic geometry II" includes content that successively continues the content of the module "Analytic geometry I", supplementing it and deepening.

Table 1. Mathematics I and Mathematics II modules

Mathematics I modules	Mathematics II modules
Analytic geometry I	Mathematical induction
Probability and statistics I	Probability and statistics II
Algebra I	Algebra II
Trigonometry I	Elements of mathematical analysis
Geometry I	Trigonometry II
	Analytic geometry II
	Geometry II

In the course Mathematics II after more than 20 years again are included elements of mathematical analysis. This crucial change is justified by a global situation since many schools in other countries teach mathematical analysis, likewise, many universities require that future students have already acquired elements of higher mathematics such as limit, derivative, integral etc.

Specific Changes in the Mathematics Standard and Curriculum for Grades 10–12

In this subsection mathematics curriculum examples for Grades 10–12 are compared, focusing on content changes in each grade:

1) previous curriculum – (Mathematics for Grades 10–12. Sample curriculum, 2008);

2) current curriculum – (Mathematics I. Sample curriculum of the optimal course for general secondary education, 2020) and (Mathematics II. Sample advanced course curriculum for general secondary education, 2020).

Previous curriculum was designed for educational programmes with 4 mathematics lessons per week. Teachers could use it as a model for their own authoring curriculum, for example, if the number of mathematics lessons per week was higher. In the current curriculum 6 lessons per week are planned for Mathematics I (Grades 10–11) and 8 lessons per week are planned for Mathematics II (Grade 12).

The General Education Law in Latvia states that the duration of a lesson in general secondary education programmes shall be 40 minutes and the duration of the academic year in general secondary education institutions shall be:

1) 35 weeks in grades 10 and 11;
2) 38 weeks in grade 12 (General Education Law, 1999).

In Tables 2, Table 3 and Table 4 are given order of topics in the previous and current mathematics curriculum, as well as number of lessons (indicated in brackets) intended for each topic. In the new curriculum is changed the emphasis on teaching approach and teaching sequence of certain mathematics topics and concepts. The main changes are commented below the tables.

Table 2. Topic Order in the Curriculum for Grade 10

	Curriculum 2008 *(4 lessons per week)*	Curriculum 2020 (Mathematics I) *(6 lessons per week)*	
	Introduction (4)		
10.1.	Vectors (10)	*Analytic geometry I*	Vectors and motion (26)
10.2.	Linear, power and quadratic functions (16)		Equation of a line (34)
10.3.	Mathematical statements, proofs (18)	*Probability and statistics I*	Combinatorics and probability (38)
10.4.	Concept of angle, triangles (16)		Statistics (30)
10.5.	Trigonometric, logarithmic, and exponent functions (20)	*Algebra I*	Rational function, algebraic fractions (46)
10.6.	Sequences (10)		Rational equations and inequalities (34)
10.7.	Algebraic expressions and equations (18)		
10.8.	Circles and polygons (16)		

In the topic "Vectors and motion" the number of lessons in the current curriculum has more than doubled. This topic is planned as an interdisciplinary topic in mathematics and physics.

In the topic "Equation of a line" equations $Ax + By + C = 0$ and $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$ and $(x - a)^2+(y - b)^2 = R^2$ are included. In the previous

curriculum only equation of a straight line in form $y = ax + b$ was included besides in topics other than analytic geometry. Physics content is also integrated in this topic.

Previously topics "Combinatorics and probability" and "Statistics" were in Grade 11. The number of lessons in the current curriculum has more than doubled. The new curriculum has more in-depth content related to elements of probability theory, e.g., students will use operations with sets to calculate the probability of events, develop an understanding of conditional probability and independence of events. The new curriculum both deepens and extends the statistical content (e.g., quartiles and box plot are included). Currently in Latvia there are no textbooks that are intended for the new standard and curriculum. In the textbooks that were used for the previous curriculum can be found materials for most of the topics, unfortunately there is a problem with the topic "Statistics", because there are no appropriate materials in Latvian that could be used in the learning process. This could have a negative effect on pupils' knowledge and the exam results.

Previously topics "Rational function, algebraic fractions" and "Rational equations and inequalities" were divided through Grade 8, 10 and 11. In the current mathematics curriculum is a topic "Rational function, algebraic fractions" that includes transformations of algebraic fractions that were previously included in the elementary school mathematics curriculum. Current content includes defining the properties of the function $y = \frac{ax+b}{cx+d}$, graphing and applying it to contexts of other learning areas such as economics.

In the previous curriculum was topic "Trigonometric, logarithmic, and exponent functions". Logarithmic functions are not included in the current curriculum for Mathematics I. The new curriculum (Mathematics I) includes only two of trigonometric functions – the sine and cosine functions which are covered in the Grade 11 topic "Sine and cosine functions".

The existing curriculum has reduced the amount of content in geometry, integrating it into the topic of trigonometry and replacing it with the topic of some analytical geometry questions. Classical geometry topics (such as angles in circles, triangles, inscribed and circumscribed quadrilateral etc.) that previously were taught in Grade 8 and Grade 10, now are included only at the end of the course Mathematics II. It is unclear the further use of this knowledge since at the universities there is no need for these concepts. However, classical geometry is a basis for mathematical olympiads starting from national level competitions till the International Mathematical Olympiad. Thus, these changes might have a negative effect on pupils' knowledge in classical geometry and cause a decrease on mathematical olympiad results, because in the olympiad problem set there always is at least one problem in classical geometry.

Table 3. Topic Order in the Curriculum for Grade 11

	Curriculum 2008 *(4 lessons per week)*	Curriculum 2020 (Mathematics I) (6 lessons per week)	
11.1.	Algebraic inequalities (18)	*Trigonometry I*	Sine and cosine functions (29)
11.2.	Geometric transformations (12)		Trigonometric expressions and equations (24)
11.3.	Elements of statistics (12)	*Algebra I*	Power with a rational exponent, geometric progression (28)
11.4.	Elements of combinatorics (12)		Exponent function (32)
11.5.	Elements of probability (12)	*Geometry I*	Straight lines and planes in space, polyhedrons (38)
11.6.	Parallel and perpendicular lines and planes (22)		Solids of revolution and combinations of solids (38)
11.7.	Trigonometric equations and inequalities (26)		
11.8.	Prism (16)		

As said above in Mathematics I (Trigonometry I) is not included functions $y = \tan x$ and $y = \cot x$. By looking only at the sine and cosine functions, in optimal level pupils might get the false impression that all trigonometric functions are bounded and continuous. Tangent and cotangent functions are included only in Mathematics II. Trigonometric inequalities also are removed from the optimal level course curriculum, simple trigonometric inequalities are included only in Mathematics II.

Topic about power with a rational exponent and geometric progression previously was in Grade 10 in different topics, but now these concepts are joined in one topic.

The new curriculum has 76 lessons for Geometry I and all polyhedrons are joined. Previously, roughly the same topics were covered in Grade 11 and 12, with 70 lessons within 5 topics.

In the previous curriculum geometric transformations was as separate topic, but now these transformations are included as part of different topics: parallel translation and axial symmetry is taught together with function transformations and in the topic “Sine and cosine functions”; rotation is in the topic “Sine and cosine functions”, but homothety is not mentioned in the current curricula in Mathematics I, although homothety is mentioned in the standard for optimal level. Geometric transformations are included as subtopic Mathematics II topic “Planimetry II”.

After many years, elements of mathematical analysis are again included in the curriculum.

Table 4. Topic Order in the Curriculum for Grade 12

	Curriculum 2008 *(4 lessons per week)*	Curriculum 2020 (Mathematics II) *(8 lessons per week)*	
12.1.	Exponential equations and inequalities (16)	*Mathematical induction (16)*	
12.2.	Logarithmic equations and inequalities (16)	*Probability and statistics II (23)*	
12.3.	Pyramids (16)	*Algebra II*	Sequences and power function (20)
12.4.	Solids of revolution (18)		Exponent function and logarithmic function (24)
12.5.	Functions (16)		Rational function and algebraic transformations (26)
12.6.	Equations and inequalities, their systems (24)	*Mathematical analysis*	Derivation and its applications (47)
12.7.	Combinations of solids (14)		Integrals and its applications (27)
12.8.	Mathematics as a value and mathematics as a tool (6)	*Trigonometry II (28)*	
12.9.		*Analytic geometry II (22)*	
12.10.		*Geometry II*	Planimetry (18)
12.11.			Stereometry (20)

These topics require that pupils can explain or define the limit of a sequence, the limit and continuity of a function, the derivative of a function, the primitive function, the indefinite and the definite integral; use derivatives and integrals of functions in mathematics and in the context of other fields of study. Emphasis is put more on practical applications of these concepts and on use of an information technology than on solving techniques and theoretical basis. The project School2030 as main arguments for the inclusion of the mathematical analysis in the secondary school curriculum states the following:

- elements of mathematical analysis are necessary for other subjects (for example, physics),
- acquisition and applications include and bind knowledge obtained before,
- it is necessary to prepare pupils for successful studies by creating an understanding of basic concepts of mathematical analysis, understanding is primary, but technique is secondary (Vilciņš, 2020).

Until now, elements of mathematical analysis were usually taught at universities during the first year of studies. Although these topics are usually taught more theoretically at universities, one should think about whether some accents must be changed in order not to duplicate what was taught at school. In addition, university teaching staff are concerned whether teachers will be knowledgeable and competent enough to teach mathematical analysis at school. Another

dangerous turn of events is that teachers without appropriate knowledge can give to pupils the wrong idea about certain mathematical concepts and their use. This concern has a reason since part of the teachers are not confident in their ability to teach mathematical analysis and emphasize that courses for teachers are necessary (Vilciņš, 2020).

Mathematics Final Exams and Changes in Exam Structure

Referring to the fact that the labour market lacks people with knowledge of STEM subjects, in school year 2008/2009, the mathematics exam was made mandatory for all secondary education pupils. Until then, pupils could choose to take or not to take a final mathematics exam, it was up to school to decide whether their pupils must take the exam or not. However, during the years 2001–2007 for all secondary education pupils it was mandatory to take a final test in mathematics (this test was easier than the exam). The year 2022 was the last year when all secondary education pupils took the same mandatory final mathematics exam.

When in the school year 2021/2022 a discussion about a mandatory final exam in one of the STEM subjects started with the main argument that it is necessary to improve pupils' knowledge in natural sciences, the Minister of Education and Science of Latvia Anita Muižniece (2021–2022) urges not to do so, if only based on the conclusions about what the mandatory mathematics exam gave (or did not give):

> *During this 12-year period, in only three years did the average score exceed the 40 percent threshold. This is not evidence of a trend, but rather a response to lower requirements, so it is clear that the mandatory exam does not automatically produce more competitive secondary school graduates. The motivation to study is not created by the requirement of exams, but by the content and quality of the process, that is why we must give a chance to the new content – only by waiting and evaluating the results of the acquisition of in-depth knowledge and skills, we can talk about the need for additional changes.* (Muižniece, 2022)

In the Table 5 are given average results in the mandatory final centralized exam in mathematics for last 14 years (Results of centralized examinations, 2022), (National examinations 2021/2022 Statistics, 2022). The year 2022 mathematics final exam can be found in (Centralized exam, Mathematics, 2022). The results in 2022 are a bit higher than in previous years, but it could be explained by the fact that before the exam it was stated what kind of tasks will be in the exam (for example, first task in the second part of the exam will be an exponential equation or inequality of the form $a^{f(x)} = a^{g(x)}$ or $a^{f(x)} > a^{g(x)}$. Thus, it was planned to reduce the negative impact of the COVID pandemic on pupils' knowledge, because Grade 12 pupils of year 2022 learned remotely for more than 1.5 years in total.

Table 5. Average Results in the Mandatory Final Centralized Exam in Mathematics by Year

Year	2022	2021	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011	2010	2009
Average, %	37.6	36.1	35.4	32.7	34.6	34.9	36.2	43.6	43.3	37.3	42.9	35.6	37.3	37.1

In 2022, Grade 11 pupils' that learn in schools and vocational education institutions and who have taken the course of the optimal level according to the new standard requirements, took the new exam for the first time. Only 4338 pupils took the mathematics exam in the optimal level and the average result in this exam was 40.03% (National examinations 2021/2022 Statistics, 2022).

According to the average score in the exams (37.6% in the mandatory final centralized exam and 40.03% in the optimal level exam), the National Centre for Education Republic Latvia concludes that the Grade 11 pupils' results are equivalent to those achieved by the Grade 12 pupils (This year, students managed to slightly improve their performance in all centralized exams, 2022).

It should be noted that only part of the pupils who learned the course Mathematics I took this exam, the rest will take it in the next school year. Thus, the average result 40.03% does not describe the situation in general, because many pupils decided to postpone their exam since it was something new and unknown, and only one sample of the exam was given. The 2022 mathematics exam sample for the optimal level can be found in (Centralized exam, Mathematics, Optimal Level, 2022).

In the Figure 4 are given overall rankings by school type in 2022 in the final exam (overall 14489 pupils) and in the optimal level exam (overall 4338 pupils). We can observe that the results for gymnasiums and secondary schools are significantly lower than in the centralized exam. One of the reasons could be that in the exam more attention was paid to correct mathematical language and explanation of the solution steps – this is a new experience and pupils do not know how much in detail they must explain their solution and judgments.

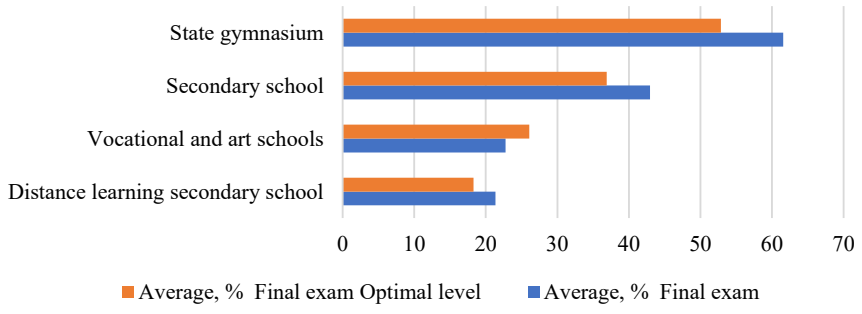

Figure 4. Overall Rankings by School Type in the Year 2022 in the Final Exams in Mathematics

Pupils who are good at mathematics frequently use facts, formulas, and algorithms that they assume are well known or obvious and do not consider it is necessary to explain anything further, but thus they can lose points. There are no precise instructions about the facts that can be used without justification, so pupils do not know what kind of solutions and explanations are expected. On the other hand, for the vocational and art schools whose pupils' results were significantly lower have improved this year. One of the reasons could be that many pupils of these schools have decided to take the final mathematics exam in the general level and only few took the optimal level exam. The other reasons could be that some very simple tasks were included or maybe it is possible that the changes in the standard and curriculum improve the basic knowledge and understanding of mathematical concepts.

Although the average results in the mathematics exam at the optimal level are higher than in this year's mandatory final centralized exam, it is difficult to predict whether this trend will continue in the following years. In addition, it should be noted that these exams are different both the structure and evaluation criteria.

Discussion and Conclusions

We have analysed how the education system in Latvia has changed over the last 30 years of independence, as well as changes in the mathematics standards and curricula since 2008.

In the school year 2022/2023 all grades (Grade 1 to 12) in all schools in Latvia are learning according to the new standard and curriculum. Compared to the previous mathematics curricula, some topics have been reordered and some topics have been moved from the primary school and some topics cover themes from the higher education. The changes in the subject content and in the teaching approach are significant, but at the moment it is not known how these changes will affect pupils' knowledge. The results of the first final mathematics exam in the optimal level show some improvement on the average score, but the increase is not significant and not all pupils took this exam.

By the new standard not all secondary school pupils learn the same content, there is an opportunity to specialize in specific fields that are necessary for the pupil in the future, also the final exam is different according to the chosen courses and subjects he acquire. Now, there are only one year's results, which were not taken by everyone who have learned the course Mathematics I, so for now there are no trends to be observed, what impact the new content and approach has on the pupils' knowledge and skills.

There is a greater emphasis on the correct use of mathematical language and these skills are evaluated in exams, this can have a negative impact on the

exam results for the talented pupils who consider many mathematical facts to be common and obvious and do not see the necessity to further justify them in more detail. Since talented pupils participate in mathematical olympiads it is not known what impact the new order of topics will have on pupils olympiad results in both national and international level.

There is no research on how the inclusion of higher mathematics elements in the secondary education will affect universities – the data are not available since first graduates that learn by the new standard will enter universities only in autumn 2023. It is possible that there will be a need for significant changes in the higher education as well.

Since the education system in Latvia is still in the process of transition there are open questions to be discussed and analysed further:

- Are all the changes justified and supported by teachers?
- Are teachers competent to teach by the new approach and the new content?
- How does the lack of textbooks affect the implementation of the content?
- How will these changes affect the study process at universities?
- How will these changes affect mathematical olympiads?

To draw qualitative conclusions about the impact of the new curriculum on the pupils learning and knowledge, it is necessary that factors affecting the learning process, such as the COVID-19 pandemic, pupils that have only partially learned following the new curriculum, preparedness of teachers, lack of textbooks, no longer have a significant impact.

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