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Primary School Mathematics Education Curricula in the United States and Latvia

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ABSTRACT

Mathematics and its related competencies are used consciously and unconsciously in many of life's everyday activities regardless of the country or geographical location, policies, social, economic or political situations. The abstract language of mathematics is understood throughout the world but is learned in a variety of different types and levels of institutions of learning. An everchanging globalized world prioritizes the need for mathematics. Therefore, the mathematics taught in school should give an understanding of mathematics and the tools to use mathematics effectively in new situations. It is common today that many countries are re-evaluating and revising their education system's standards for mathematics teaching, recognizing the changing needs of the work force and society. Within the framework of a project implemented at the Faculty of Pedagogy, Psychology and Art at the University of Latvia, one of the main priorities of which is to promote the exchange of academic staff and cooperation in training future teachers, it was possible to evaluate and compare mathematics curricula in both countries. The aim of this paper is to do a preliminary analysis of the content of the basic education curricula in Latvia and U.S., focusing specifically on geometry and measurement, and seek data about the impact of the Standards on learning. The study, using document analysis, reviews the competency-based approach taken in Latvia's Skola2030 (School2030), and compares it to the Common Core State Standards, implemented in the U.S. A review of the similarities and differences in the content and sequencing is explored. Reviewing the mathematics content in both countries showed more similarities than differences in geometry and measurement. The study looked at not only the mathematical content of the standards but also the approach both countries' standards take in developing students' conceptual understanding of primary mathematics to promote mathematical literacy for all students.

Keywords: content, curricula, mathematics education, mathematics standards, primary school

Introduction

Many processes connected to education change along with societal shifts. Accordingly, laws and regulations regarding mathematics education are affected by educational, philosophical, and political positions. Mathematics teaching is often regulated by some form of a governing document through a curriculum or a set of standards (Ernest, 1991; Niss, 1981). Stakeholders in this process have a great interest in seeing the intentions behind the standards or curricula influence learning outcomes (Ellsworth, 2000; Boesena et al., 2014). Regardless of the country or geographical location, policies, social, economic or political situations, countries are reevaluating and revising their national standards for mathematics education, recognizing the changing needs of the work force and society. Latvia and the United States, as many countries worldwide, have sought to improve the quality of their education systems, especially in the STEM fields. Many countries have updated their standards or curriculum guidelines to promote systemic changes in their respective compulsory education programs. As examples, in the U.S. Common Core State Standards (Common Core State Standards [CCSS], 2010) were developed, but Latvia introduced its new mathematics program School 2030 (Skola2030, 2018).

In order to assure the effectiveness of new standards, these curriculum guidelines need to be designed and developed within evidence-based instructional practice, monitoring progress and making data-informed instructional decisions (Fuchs & Fuchs, 2007; Rinaldi & Samson, 2008). Traditional curriculum focused little on the practices of doing mathematics but rather on mathematical structures: notions, concepts, theories, methods, and results (Hoffmann, 1989). During the 1990s changes emerged. Recently developed curricula aim to clarify the relationship between mathematical content and practice and make the development of sound mathematical practice an explicit curricular goal (Boesena et al., 2014):

> *Every researcher, every producer of mathematics will readily admit that mathematics is an activity* (Freudenthal, 1991, p. 14).

The classroom or other learning space is no longer a place where the teacher imparts his or her knowledge to students who are expected to listen passively to the teacher and wait for him or her to tell them what to do. In modern times, it is expected that students are active participants in the learning process (Bada, 2015). Today's student needs to change from a passive participant in the learning process, who quietly listen to the teacher's narration, to an active participant in the learning process, who brings with his own learning experiences and approaches to learning (Rajendra, 2019; Thompson, 2015). Teachers need to recognize that mathematics and mathematics teaching must be dynamic (Machisi, 2021). Many educators agree that the mathematics classroom must be

an environment that gives students the opportunity to be actively involved in learning, not just passive receivers of information. Thus, in documents regulating and determining educational processes, including standards and programs, it is necessary to specifically include language that promotes the active involvement of students in learning mathematics. It is thus worthy to ask how standards differ in content and structure. A closer analysis of similarities and differences may lead to discovering paths to improvement in both countries.

Methodology

The aim of this paper is to do a preliminary analysis of the content of the basic education curriculums in Latvia and U.S., focusing specifically on geometry and measurement, and seek data about the impact of the standards on learning. The study was conducted between January 2022 and August 2022.

International assessments, like the Trends in International Mathematics and Science Study (TIMSS) provide opportunity for countries to identify strengths and areas of improvement in curriculum. This study compared primary school mathematics standards in Latvia and the United States in the context of the 2019 4th grade TIMSS results. Overall achievement on the TIMSS assessment has increased over the years in both countries and both are doing well by international comparison. In 2019 Latvia ranked 10th best in 4th grade mathematics while the United States was 15th of 58 countries. Content domain specific results are shown in Table 1 (Mullis et al., 2020).

Table 1. Trends in International Mathematics and Science Study (Mullis et al., 2020)

Country	Average Score			
	Total	Number	Data	Measurement and Geometry
Latvia	546	547	542	548
United States	535	542	533	520

Reviewing the content domains overall, Latvia and the U.S. had similar scores in Number and Data, but the largest difference was found in Measurement and Geometry (Table 1). Based on this, the focus of this study was placed on this content domain. The difference between the score in Measurement and Geometry was significantly different than the overall mathematics average scale score for the U.S., indicating an area for improvement.

The study, using document analysis, selected the basic education curricula for mathematics currently used in the United States and Latvia (Table 2). Documents that meet the following criteria were selected for the document analysis: intended for primary school education, define mathematics content and learning outcomes, and are issued in the period from 2010 till 2022. The curricula were analyzed by

using content analysis as a research method. The learning outcomes tables was structured to illustrate the document content and describe the different relationships between learning outcomes that make up the text.

Table 2. Analyzed Mathematics Curricula

Title	Publisher	Year	Country
Mathematics 1.–9. grade	National Centre for Education (VISC)	2018	Latvia
Regulations Regarding the State Basic Education Standard and Model Basic Education Programmes No. 747	Cabinet of Ministers	2018	Latvia
Common Core State Standards (CCSS)	National Governors Association for Best Practices and Council of Chief State School Officers	2010	U.S.

Standards alone cannot change what is taught. Real changes to what is taught and how it is taught is dependent upon implementation of new standards. While it is impossible to say with absolute certainty that they are, standards that are reflected in student textbooks are a strong indication that to some degree the standards are implemented. Research has shown that textbooks play a major role in determining what is to be taught and the experiences students encounter (Gene, 2018). Textbooks are a tool that translate standards into classroom interactions. They are a resource that are used to provide opportunities to master the knowledge and skills that have been identified as important by the education system. Textbooks have been found to strongly affect teachers' instruction in multinational studies (Valverde et al., 2002).

To begin the study, three popularly used U.S. textbook series were chosen as claiming to adhere to the Common Core State Standards (CCSS, 2010). The textbook series included:

- Common Core Progress Mathematics published by Sadlier School,
- McGraw-Hill MyMath published by McGraw-Hill,
- EnVision Mathematics published by Scott Foresman.

The individual textbooks for 1st through 4th grade for each series were reviewed to ascertain the presence of content for each standard. It was found that this was indeed the case with few exceptions. Thus, it can be reasonably assumed that many schools in the United States are actually teaching material required by the standards at that grade level. In Latvia, the Ministry of Education and Science, which certifies that a textbook adheres to the country's current standards, examine textbooks. While Latvia's current standards were revised and adopted in 2018, the 2006 standards are predecessors to the current standards

and appear in textbooks used throughout the country. The mathematics textbooks for primary school included different publishers:

- Mathematics 1st – 3rd grade published by Lielvards,
- Mathematics 1st – 3rd grade published by Zvaigzne ABC,
- Mathematics 1st – 3rd grade published by Petergailis.

The Latvian standards are actually law governing the education system. The United States has for the most part adopted the Common Core State Standards (CCSS, 2010) but it is not law. The CCSS were sponsored by the National Governors Association and the Council of Chief State School Officers. In the United States, the regulation of schools is governed by state or territory legislature, not the federal government. Thus, states had the option to adopt or reject CCSS. Over 40 states and territories adopted Common Core since its release in 2010 as its fundamental document for guiding what children in grades K-12 learn and grade level goals in mathematics and English (CCSS, 2010).

The current standards for both Latvia and the United States were reviewed for overall structure and mathematical content in first through third grade. It is important to note that Latvian school children begin 1st grade at age 7, while American children at age 6. Thus, it was appropriate to also review 4th grade for American content. The focus was on the geometry and measurement standards of both countries. Measurement was limited to measurement associated with geometry; length, area and volume.

Results

Reviewing the mathematics content in both countries show more similarities than differences in geometry and measurements. The United States Common Core State Standards (CCSS, 2010) do not specify how the content is to be taught or the sequencing of the material. Integral to CCSS are the first eight Mathematical Practices Standards. These standards are overarching over all content and considered to be an essential part of what it means to be mathematically literate. They include:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning (CCSS, 2010).

Mathematics learning standards in Latvia and the adopted program School 2030 (Matemātika, 2018) are more numerous and are more precise and detailed

in their descriptions and stress both knowledge and skill goals. School2030 (2018) includes both a suggested sequencing of each major topic as well as methodological commentary. The learning content in mathematics is organized according to the Big Ideas (*Lielās idejas*) that the student has to acquire in order to develop common understanding about the surrounding world and oneself in it. The big ideas form the structural framework of the compulsory learning content. Requirements for the acquisition of the learning content or the learning outcomes that the pupil has to attain finishing the particular stage of education are described according to these big ideas: mathematics language; strategies and reasoning in mathematics; numbers, operations with numbers, algebra and functions; shapes; data and statistics (Matemātika, 2018). The learning outcomes are defined both for each theme in the learning content and the respective educational three-year period, finishing Grade 3, 6 and 9.

The standards in both countries state specific learning goals but the Latvian standards give a more precise and detailed description of desired learning outcomes. For instance, the learning outcomes for geometry in the Latvian standards and accompanying program divide geometry into categories that encourage solving practical problems and encourage students to form conclusions about shapes and space:

- shapes and their components and properties;
- position of figures in a plane, in space and their mutual position;
- equality and similarity of shapes;
- movements and transformations of shapes in the plane (Regulation No. 747, 2018).

It specifically delineates skills and gives emphasis to conceptual understanding. Latvian standards contain much more material for the teacher. It could be argued that the verbosity may hinder some but gives more detailed information and guidance to those who seek it.

Table 3 below gives an abbreviated version of the Measurement and Geometry standards for Grades 1–3 regarding mathematical content for both countries side by side. Similar standards were grouped side by side for comparison. The table includes the action words used in each standard, emphasizing student engagement. The U.S. 4th grade standards include: convert linear units of measurement; apply area and perimeter formulas; recognize and draw parallel and perpendicular lines, angles and symmetry.

To ascertain whether there were connections between differences in standards and differences in achievement, individual TIMSS test questions were examined. TIMSS releases only a small number of test questions. In Measurement and Geometry, three questions were released from the 2019 4th grade study. One item is classified as an Intermediate International Benchmark of Mathematics Achievement item, while the other two are Advanced (Mullis et al., 2020).

Table 3. Results of Geometry in 1st Grade, 2nd Grade, 3rd Grade

Grade	Mathematics by School 2030 (2018) Latvia	Common Core State Standards (2010) U.S.
1st Grade	**Compose** 2-d and 3-d shapes from smaller shapes **Compare** lengths **Compare** attributes of various shapes **Draw** shapes with straight-edge and free-hand **Measure** lengths **Divide** shapes into two and four equal pieces	**Compose** 2-d and 3-d shapes from smaller shapes **Compare** lengths, order 3 lengths **Distinguish** defining attributes of shapes Build and draw shapes with defining attributes **Express** a length by unit lengths **Partition** 2-d shapes into 2 and 4 equal pieces
	Build 2-d and 3-d shapes with stick-like materials **Recognize** symmetry in figures **Draw** symmetric images **Group** shapes by properties **Decompose** shapes in various ways **Make** ethnic straw ornaments **Recognize** congruent shapes **Duplicate** by drawing a given shape on graph paper **Draw** lines of given cm length	
2nd Grade	**Name** polygons **Draw** shapes with given properties **Cover** shapes with given shapes **Decompose** shapes in different ways, discuss all possibilities **Divide** rectangles into equal squares and other figures in different ways **Divide** a figure into 2 equal parts different ways	**Name** polygons **Measure** lengths using different units and measuring instruments **Estimate** lengths **Compare** lengths **Recognize** and **draw** shapes with specific attributes **Partition** a rectangle into same-sized squares and count total **Partition** circles and rectangles into 2, 3 and 4 parts **Recognize** that equal shapes of identical wholes need not have the same shape
	Compose 3-d shapes from cubes **Describe** shapes from various perspective **Draw** given cm or mm length **Mark** halves and quarters of a segment **Compute** perimeter **Find** area as covering by a given number of unit squares **Draw** rectangles with given perimeter or area on graph paper	

Table 3. Continued

3rd Grade	**Measure** lengths using different units and measuring instruments **Estimate** lengths **Compare** lengths **Recognize** and **draw** shapes with specific attributes **Partition** a rectangle into same-sized squares and count total **Partition** circles and rectangles into 2, 3 and 4 parts **Recognize** that equal parts of identical wholes need not have the same shape	**Understand** concept of unit square and area **Measure** halves and fourths of an inch **Measure** area by counting and multiplication of side lengths, connect these concepts **Decompose** areas into smaller areas **Find** perimeter of various shapes **Exhibit** rectangles with the same perimeter and different areas and different perimeter and same area **Understand** classes of shapes can have common attributes **Draw** quadrilaterals that do not belong to special subcategories **Partition** shapes into parts with equal area
	Explore different types of angles **Make and draw** shapes with given angles or sides **Describe** and **identify** faces, edges, vertices **Make** 3-d shapes with stick-like materials and with nets **Explore** nets of solids **Build** right rectangular parallelepipeds with cubes **Make** 2-d scale drawings of real-life objects **Compute** volume of right rectangular parallelepiped by counting cubes	

Intermediate test question 1.11.1 asks students to draw the symmetric image of a 2-dimensional shape. Results show that 89% of Latvian 4th graders correctly answered the question while only 60% of students in the United States. The international average on this item was 70%. Latvian students scored significantly better than the average while U.S. students significantly lower (Mullis et al., 2020).

To help explain this large achievement gap, we note that in Latvia, School 2030 (Matemātika, 2018) introduces symmetry in 1st grade. Students are asked to recognize reflective symmetry and draw the image for themselves. In contrast, Common Core State Standards (2010) introduces the idea of symmetry in 4th grade but does not specify that students should actually create the image themselves but rather to simply identify the line of symmetry. While symmetry in primary school standards can be argued as non-essential, it shows the importance of standards in student learning.

Another difference was the inclusion of describing shapes from various perspectives in the Latvian 2nd grade standards. This is not included in U.S. standards through 4th grade. It should also be noted that Common Core State Standards (2010) does not mention drawing lines of a certain length to be a goal, but rather just to measure. This skill is specifically mentioned in School 2030 (Matemātika, 2018). Several content differences become apparent also in 3rd Grade. For example, another goal in the Latvian Standards is to make scale drawings of real-life objects.

Advanced question 1.13.2 asks students to fill a 6 × 6 cm square using multiple copies of three different shapes: 3 × 3 squares, 2 × 6 rectangles and 6 × 6 right triangles. Students need to identify how many copies of each of the three shapes are needed to cover the square. On this question, the percentage of Latvian students who answered the question correctly was almost double the U.S. students (31% vs 17%) (Mullis et al., 2020). In this question again Latvian students scored significantly better than the TIMSS average while U.S. students significantly lower. Latvian standards pay specific attention to the concept that equal parts may not have the same shape.

In advanced test question 1.13.3 students are shown three-dimensional shapes and asked how many triangles and squares faces they see in each shape. Latvian students scored significantly lower than the international average with 22% of students answering the question correctly. Students in the U.S. did better on this question with 25% answering correctly which is below the international average of 27%, but not significantly (Mullis et al., 2020). This slight advantage might be due to the emphasis the U.S. standards place on identifying, rather than building for themselves.

Conclusions

- Mathematics standards can have varying degrees of importance in regards to oversight by governing institutions. The Latvian standards are law governing the education system and the United States has for the most part adopted the Common Core State Standards but it is not federal law. The level of detail and components of the standards also vary.
- Both countries standards address not only the mathematical content that should be mastered at each grade level, but also emphasize the opportunity for students to be actively involved in the mathematics acquisition process.
- Minor differences appear in geometry and measurement between the standards of Latvia and the U.S. There are differences in the sequencing of geometric content as well as skill acquisition.
- Textbook adherence to standards in Latvia is certified by the government, but there is no such oversight in the U.S.

- While these differences may appear small, focus on such nuances could help both countries improve test scores and most importantly help their students gain a better understanding of mathematics.
- Reviewing results on international assessments such as TIMSS points out the importance of the standards and even the wording of standards, indicating that well thought-out changes to standards can have an impact on student knowledge.

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